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METHODS OF INVESTIGATING THE DYNAMICS OF ROCKET CARRIERS AND SPACE VEHICLES

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Translation of "Metody issledovaniya dinamiki raket-nositeley i kosmicheskikh apparatov".
Paper presented at the 2nd Annual Conference of the American Astronautical Society, 4 - 7 May 1965, Chicago.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON MAY 1965

NASA TT F-9544

METHODS OF INVESTIGATING THE DYNAMICS OF ROCKET CARRIERS AND SPACE VEHICLES **/1

G.N.Mikishev*

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The dynamics of rocket carriers and Space vehicles is investigated on the example of an absolutely rigid body having cavities partially filled with liquid, to take disturbed motion produced by propellant sloshing into consideration. Theoretical and experimental methods for determining the hydraulic factors for cavities of various shapes (cylindrical, spherical, conical, and toroidal) are discussed, with a comparative analysis of the results and instructions for programming the data on digital computers. The stabilization problem is solved on an analog computer, feeding the dynamic properties of cylindrical cavities over a mechanical model and writing the equations of disturbed motion into the computer.

author

INTRODUCTION

Modern rocket carriers and space vehicles carry large mobile masses of liquid propellant with free surfaces and have shells that may possess considerable elasticity.

These features impose serious and often contradictory restrictions on the choice of the parameters for the stabilization system, making it difficult to design such a system without modifying the entire design and assembly (changing the centering, use of special fluid vibration dampers, etc.).

Insufficient emphasis on these points may have unfortunate effects on the stability of motion, on the operating reliability of the control system, and on the strength of the shell. These aspects must therefore be taken into account in designing stabilization systems for such objects, which makes it necessary to investigate the dynamic stability of highly complex mechanical systems. In the general case, the system that must be considered comprises an elastic body with cavities partially filled with a liquid. In the past few years, problems of the dynamics of such systems have received considerable attention in the USSR and in other countries (N.N.Moiseyev, G.S.Narimanov, D.Ye.Okhotsimskiy, B.I.Rabinovich, Abramson, Bauer, Miles, and others).

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**Numbers in the margin indicate pagination in the original foreign text.

This paper deals only with the clearly delimited field of problems connected with the dynamics of an absolutely rigid body having cavities partially filled with a liquid. A large portion of the paper is devoted to the theoretical and experimental methods of determining the hydrodynamic coefficients of the equations of disturbed motion for cavities of various shapes, with special reference to a comparative analysis of the results obtained by various methods. For illustrative purposes, computational and experimental data for cylindrical, spherical, conical, and toroidal cavities are included. All calculations were performed on electronic computers. We also discuss several problems of the stability of motion. /2

1. Fundamental Equations

Consider the disturbed motion of a solid with a cavity partially filled with a liquid. The generalization of the results given below to the case of an arbitrary number of cavities is trivial. We introduce the body-fixed coordinate system Oxyz which, in undisturbed motion, coincides with the system O*x*y*z*. Let us assume that the coordinate system O*x*y*z* is subject to the action of potential mass forces with a gradient $\bar{J}$, collinear with the direction of the axis O*x*. We also assume that the fluid is ideal and incompressible.

The generalized coordinates and velocities characterizing the disturbed motion will be assumed to be small quantities, i.e., we start from the conventional assumptions of the theory of small wave motions of an ideal, incompressible fluid.

The viscosity of the fluid at high Reynolds numbers which is of the most practical interest is taken into account, in first approximation, by introducing, into the equations of wave motion of the fluid, terms proportional to the corresponding generalized velocities. These terms will then be introduced into the equations of disturbed motion. /3

Under the adopted assumptions, a system of equations of disturbed motion can be obtained. As the initial equations of disturbed motion, let us take the equations for a body with a fluid having two planes of symmetry Oxy and Oxz relative to the metacenter G, whose coordinate x_G is defined in terms of the coordinate x_T of the center of mass of the solid-liquid system by the formula

$$x_G = x_T + \frac{\rho\Omega}{m^\circ + m}. \tag{1}$$

The equations are of the following form:

$$\begin{gathered}(m^\circ + m)\ddot{u} + \sum_{n=1}^{N} \lambda_n \ddot{\eta}_n = P,\\ (J^\circ + J)\ddot{\psi} + \sum_{n=1}^{N} \lambda_{on} \ddot{\eta}_n = M_y,\end{gathered} \tag{2}$$

$$\mu_n(\ddot{r}_n + \varepsilon_n \dot{r}_n + \omega_n^2 r_n) + \lambda_n \ddot{u} + \lambda_{on}\ddot{\psi} = 0$$
$$(n = 1, 2, \ldots N)$$

$$(I^\circ + I)\ddot{\varphi} + \sum_{m=1}^{M} \lambda_{xm}\ddot{q}_m = M_x$$

$$\gamma_m(\ddot{q}_m + \varkappa_m \dot{q}_m + \sigma_m^2 q_m) + \lambda_{xm}\ddot{\varphi} = 0. \qquad (3)$$
$$(m = 1, 2, \ldots M)$$

where the following notations are used:

- $(m^\circ + m)$ = mass of the entire solid-liquid system;
- $(J^\circ + J)$, $(I^\circ + I)$ = reduced moments of inertia of the system about the axes G_y and G_x;
- P = external force acting in direction of the axis G_z;
- M_y, M_x = external moments about the axes G_y and G_x;
- u, ψ, φ = displacement of the coordinate system Gxyz in the plane $0^*x^*z^*$ and its rotation about the axes G^*y^* and G^*x^*;
- r_n, q_m = generalized coordinates of the oscillating fluid excited on motion in the plane $0^*x^*y^*$ and rotation about the axis G^*x^*;
- N, M = number of degrees of freedom to be taken into account for the oscillations of the fluid;
- ρ = mass density of the fluid;
- Ω = equatorial moment of inertia of the free upper surface of the liquid Σ.

The equations of disturbed motion in the plane $0^*x^*z^*$ are analogous in form to the equations of motion in the plane $0^*x^*y^*$.

The hydrodynamic coefficients entering into eqs.(2) and (3) are expressed in terms of the corresponding dimensionless coefficients in the following manner:

$$\lambda_n = \rho\ell^3\bar{\lambda}_n, \quad \lambda_{on} = \rho\ell^3\left[\ell\bar{\lambda}_{on} - (x_0 - x_G)\bar{\lambda}_n\right],$$

$$\mu_n = \rho\ell^3\bar{\mu}_n, \quad \omega_n^2 = \frac{g}{\ell}\bar{\omega}_n^2, \quad \varepsilon_n = \frac{\delta_n\omega_n}{\pi}, \quad \varkappa_m = \frac{\beta_m\sigma_m}{\pi}, \qquad (4)$$

$$\gamma_m = \rho\ell^3\bar{\gamma}_m, \quad \lambda_{xm} = \rho\ell^4\bar{\lambda}_{xm}, \quad \sigma_m^2 = \frac{g}{\ell}\bar{\sigma}_m^2, \quad I = \rho\ell^5\bar{I},$$

$$J = \rho\ell^5\bar{J} + m\left[(x_D - x_G)^2 - (x_0 - x_D)^2\right],$$

where

ℓ, x_0 = characteristic dimension and coordinate of the characteristic point;

m, x_0 = mass of the fluid and the coordinate of its metacenter; /5

δ_n, β_n = logarithmic decay of oscillation of the fluid.

The dimensionless coefficients are defined by the formulas:

$$\lambda_n = \int_S \varphi_n \nu_z \, dS = \int_\Sigma z \frac{\partial \varphi_n}{\partial \nu} dS, \qquad \bar\lambda_{on} = \int_S \varphi_n (z\nu_x - x\nu_z) \, dS = \int_\Sigma \Psi \frac{\partial \varphi_n}{\partial \nu} dS,$$

$$\bar\gamma_n = \bar\mu_n = \int_\Sigma \varphi_n \frac{\partial \varphi_n}{\partial \nu} dS, \qquad \bar J = \int_S \Psi (z\nu_x - x\nu_z) \, dS, \tag{5}$$

$$\bar\lambda_{xm} = \int_S \varphi_m (y\nu_z - z\nu_y) \, dS = \int_\Sigma \Phi \frac{\partial \varphi_m}{\partial \nu} dS, \qquad I = \int_S \Phi (y\nu_z - z\nu_y) \, dS$$

where S and Σ are the wetted surface and the free surface of the liquid; ν_x, ν_y, ν_z are the coefficients of the unit vector of the exterior normal to the surface S + Σ. The functions φ_n, Ψ and Φ are the solutions of the boundary problems:

$$\Delta \varphi_n = 0, \quad \frac{\partial \varphi_n}{\partial \nu} = 0 \text{ on } S, \qquad \frac{\partial \varphi_n}{\partial \nu} = \omega_n^2 \varphi_n \text{ on } \Sigma,$$

$$\Delta \Psi = 0, \quad \frac{\partial \Psi}{\partial \nu} = (z\nu_x - x\nu_z) \text{ on } S, \qquad \frac{\partial \Psi}{\partial \nu} = 0 \text{ on } \Sigma, \tag{6}$$

$$\Delta \Phi = 0, \quad \frac{\partial \Phi}{\partial \nu} = (y\nu_z - z\nu_y) \text{ on } S, \qquad \frac{\partial \Phi}{\partial \nu} = 0 \text{ on } \Sigma$$

The equations of disturbed motion [eqs.(2)] are rather complicated. The results of numerous theoretical and experimental studies indicate that it is sufficient to use the first form of oscillation of the fluid, which plays a special role in the development of transients in dynamic systems of this type and determines their stability (in the case of simply-connected and doubly-connected cavities without partitions).

In cavities of revolution with radial partitions, the fundamental frequency and the first harmonic of the oscillations of the fluid may be equivalent, so that both must be taken into account rather than the fundamental frequency alone.

Below, we will confine the calculation to simply-connected and doubly- /6

connected cavities of revolution and therefore will identify the fluid in each cavity with a system with one degree of freedom, neglecting all higher harmonics.

The equations of disturbed motion in the plane G*x*z*, allowing for the existence of a stabilizing system and described by operators which in the general case are nonlinear, then take the form:

$$\left.\begin{aligned} &(m^\circ + m)\ddot{u} + \lambda_1\ddot{\eta}_1 = P(\delta), \\ &(J^\circ + J)\ddot{\psi} + \lambda_{01}\ddot{\eta}_1 = M_y(\delta), \\ &\mu_1(\ddot{\eta}_1 + \varepsilon_1\dot{\eta}_1 + \omega_1^2\eta_1) + \lambda_1\ddot{u} + \lambda_{01}\ddot{\psi} = 0, \end{aligned}\right| \tag{7}$$

$$L(\delta) = A(u) + B(\psi). \tag{8}$$

2. <u>Determination of the Hydrodynamic Coefficients</u>

The hydraulic factors may be determined by theoretical or experimental methods.

The theoretical methods are widely used and well known. They include, for cylindrical cavities with a flat bottom, the method of separation of variables; for noncylindrical cavities, the Ritz variational method as well as the methods of the theory of long waves.

For cylindrical flat-bottomed cavities, the boundary problems are directly solved by the method of separation of variables. As an example, we give the solution of the boundary problem for the displacement potential of wave motions of the liquid, obtained by this method for a cavity formed by coaxial cylinders with radial partitions: /7

$$\varphi_n = \frac{ch(\xi_n x)}{\xi_n sh(\xi_n h)} \frac{J_\nu(\xi_n R)N'_\nu(\xi_n) - N_\nu(\xi_n R)J'_\nu(\xi_n)}{J_\nu(\xi_n)N'_\nu(\xi_n) - N_\nu(\xi_n)J'_\nu(\xi_n)} \cos\nu\left(\eta + \frac{\pi}{k}\right), \tag{9}$$

where

k = number of partitions, $\nu = \frac{\pi k m}{2}$, $m = 0, 1, 2, \ldots;$

ξ_n = root of the equation, $J'_\nu(\delta\xi)N'_\nu(\xi) = J'_\nu(\xi)N'_\nu(\delta\xi)$;

δ = ratio of the radii of the inner and outer cylinders;

x, R, τ = cylindrical coordinates.

Expressions for the displacement potential can be derived from eq.(9) for the cavities in the form of a cylinder, a cylinder with radial partitions, or

coaxial cylinders, considered by G.S.Narimanov, D.Ye.Okhotsimskiy, and Bauer.

The Ritz variational method has proved to be the most flexible and reliable method for solving the boundary problems for cavities of widely varying shapes. When using the variational method, the boundary problems (6) reduce to a minimizing of certain functionals, which is accomplished by the direct Ritz-Trefftz methods.

The application of the long-wave theory is limited to the case of small depth of the liquid. The method of equivalent cylinders inscribed in the perimeter of the free surface may also be used for approximate estimates.

Experimental methods play an important part in these problems. They permit determination of the hydraulic factors in cases where the use of theoretical methods encounters serious difficulties, for example, when there are various types of discontinuous partitions in the cavity; such methods also give a criterion for the reliability of the numerical results obtained by approximate calculation methods. Very few literature data are available on experimental work on this subject matter; almost all relate to the special case of deter- /8 mining the natural frequencies and oscillation modes of the liquid together with the damping factors. For this reason, we will describe one of the experimental methods in more detail. Let us use eqs.(7) as basis.

The natural frequencies and damping factors are generally determined by the method of free or forced vibrations. The remaining coefficients can be determined by various methods: by measuring the natural frequencies of the subsystems corresponding to the translational and rotational motion of the cavity; by measuring the forces and moments exerted by the fluid on the cavity; by plotting the frequency characteristics and the oscillation modes of the fluid, etc. The first of these methods, in many respects, is the most convenient.

Let us consider separately the two subsystems describing the displacement of the body (ψ = 0) and its rotation about the metacenter of the solid-liquid system (u = 0), assuming for simplicity that

$\varepsilon_1 = 0$:

$$\begin{aligned} &(m^\circ + m)\ddot{u} + \lambda_1 \ddot{\eta}_1 = 0, \\ &\mu_1(\ddot{\eta}_1 + \omega_1^2 \eta_1) + \lambda_1 \ddot{u} = 0, \end{aligned} \tag{10}$$

and

$$\begin{aligned} &(J^\circ + J)\ddot{\psi} + \lambda_{01} \ddot{\eta}_1 = 0, \\ &\mu_1(\ddot{\eta}_1 + \omega_1^2 \eta_1) + \lambda_{01} \ddot{\psi} = 0. \end{aligned} \tag{11}$$

From the characteristic equations of the subsystems [eqs.(10) and (11)], the following two relations can be derived:

$$\frac{\lambda_1^2}{\mu_1 (m^\circ + m)} = 1 - \frac{\omega_1^2}{\omega_u^2}, \tag{12}$$

$$\frac{\lambda_{01}^2}{\mu_1 (J^\circ + J)} = 1 - \frac{\omega_1^2}{\omega_\psi^2}, \qquad (13)$$

whose left-hand sides, in the case of geometric similarity of the cavities /9
(when the corresponding similarity holds for the parameters m° and J°), remain constant at a prescribed liquid level, i.e., are invariants. The invariants play an important role in the investigation of the dynamic stability of a solid, filled with liquid.

On the basis of eqs.(12) and (13), the invariants are readily determined by measuring the frequencies ω_1, ω_0 and ω_ψ. At known values of $(m^\circ + m)$ and $(J^\circ + J)$, we can derive the invariant dynamic characteristics $\frac{\lambda_1^2}{\mu_1}$ and $\frac{\lambda_{01}^2}{\mu_1}$, connected only with the shape of the cavity.

To determine the individual coefficients, two more conditions must be found in addition to the two relations considered above. Prescribing a translational displacement of the cavity in accordance with the harmonic law $u = u_0 \sin pt$, we find, from the second of eqs.(10),

$$\frac{\lambda_1}{\mu_1} = \frac{r_{01}(p)}{u_0}\left(\frac{\omega_1^2}{p^2} - 1\right), \qquad (14)$$

where $r_{01}(p)$ is the amplitude of the liquid at the wall, for the prescribed oscillation frequency p of the cavity ($p < \omega_1$).

Introducing, additionally, the elastic moment $c\psi$ in the first equation of the system (11), it is easy to obtain one more relation for determining the conjugate moment of inertia:

$$(J^\circ + J) = \frac{c\left(1 - \frac{\omega_\psi^2}{\omega_1^2}\right)}{\omega_\psi^2\left[1 - \frac{\omega_\psi^2}{\omega_1^2}(1 - \Delta_0)\right]}, \qquad (15)$$

where $\Delta_0 = \frac{\lambda_{01}^2}{\mu_1 (J^\circ + J)}$ is found from eq.(13) and ω_ψ is the natural frequency of the system.

Thus, the relations (12)-(15) obtained in this manner permit to determine the required coefficients λ_1, λ_{01}, μ_1, and J.

Either the actual cavities or geometrically similar models of the cavities /10
may be used to determine the hydraulic factors, the invariant dynamic characteristics, and the natural frequencies. The model of the cavity must not only satisfy the requirements of complete geometric similitude, but also the re-

striction of weight and inertial characteristics.

For the model of the liquid, no particular restrictions are imposed. The failure of complete similitude with the Reynolds and Bond numbers in the problem under consideration has no special significance. The influence of these factors might almost vanish for very small models.

The translational and angular motions of the cavity, described by the subsystems of equations (10) and (11), are realized in practice by the aid of the devices schematically shown in Fig. 1 b, d.

For both translational and angular motion of the cavity, the natural frequencies ω_1, ω_u and ω_ψ are measured with an error not exceeding 1 - 2%. Consequently, such important characteristics as $\frac{\lambda_1^2}{\mu_1}$ and $\frac{\lambda_{01}^2}{\mu_1}$ may be determined with high accuracy.

The translational harmonic oscillations of the cavity, for determining the ratios of the coefficients, may be reproduced by the aid of a kinematic test stand. The error of determination of the ratio λ_1/μ_1 depends primarily on the error of measurement of the oscillation amplitudes of the liquid. As a rule, this error does not exceed 5 - 7%. The error of determination of the conjugate moment of inertia is often somewhat greater than the error of determination of the ratio λ_1/μ_1.

This experimental method yields satisfactory results if the damping of the oscillations of the liquid (logarithmic decrement) does not exceed 0.2 - 0.3, /11 i.e., in the great majority of practical cases.

The dimensionless hydraulic factors have been tabulated for a large number of cavities of widely varying shapes.

We will consider only a few calculation results and experimental data in determining the dimensionless hydrodynamic factors for various cavities. For each such cavity, we derive the invariant dynamic characteristics ω_1^2, $\frac{\lambda_1^2}{\mu_1}$, $\frac{\lambda_{01}^2}{\mu_1}$, and J which play the most important role in dynamic studies. The cavities were selected on the basis of the assumed cavities of elementary geometric configuration (cylindrical, spherical, conical, and toroidal). Some data are also given for a cavity of more intricate configuration, namely a circular cylinder with radial fins.

All calculations were mainly performed by the variational method on a digital computer. Special emphasis was placed on the selection of coordinate functions (usually, they were harmonic functions satisfying the greatest possible number of boundary conditions). For comparison we also will give some data taken from work by other authors.

Coaxial cylinders. A solution of the problem of determining the hydrodynamic factors for a cavity formed by two coaxial cylinders with a flat bottom is obtained as a special case of eq.(9). Figures 2 - 5 show the results of calculations and experimental determination of the invariant dynamic characteristics for the fundamental frequency of the oscillations of the liquid as a function of the depth of the liquid and the ratio of the radii of the cylinders. The experimental points lie directly on the calculated curves.

Cylinder with radial fins. Consider a cavity in the shape of a right circular cylinder with a flat bottom and equidistant radial fins along its walls to a height h_0. The width of each fin will be denoted by b. /12

There exists no theoretical solution for this particular problem. We therefore give the experimental results. The dynamic characteristics ω_1 and $\frac{\lambda_1^2}{\mu_1}$ were plotted against the dimensionless parameters $\bar{h}$ and k for k = 1.3 (Figs.6 and 7). It is obvious from these graphs that the presence of fins in the tank may result in substantial changes in the hydrodynamic characteristics.

Sphere. The problem of the oscillations of a liquid in spherical cavities has been considered by a number of authors, although for the most part they confined their investigations to a determination of the natural frequencies (Budianskiy, Abramson, etc.).

Figure 8 shows the coefficients ω_1^2 and $\frac{\lambda_1^2}{\mu_1}$ calculated by the variational method using spherical and cylindrical coordinate functions, and the same coefficients for a cylindrical cavity of the same depth and radius as the current values for the spherical cavity. All data are relatively close together, but those corresponding to the spherical system of coordinate functions are the most exact and are therefore to be preferred, although the convergence of the minimizing sequences is somewhat poorer in this case.

The theory is in satisfactory agreement with the experiments throughout the range of depths. It also agrees with Budianskiy's results for $\bar{h} \to 0$, $\bar{h} = 1$, $\bar{h} \to 2$.

Cone. The principal hydrodynamic characteristics were also determined for an upright cone and an inverted cone. For the inverted cone, only one parameter, the vertex half-angle θ, had to be determined. The numerical values were found by the calculus of variation, using a specially constructed system of "conical" functions, which were generalized spherical Legendre functions of nonintegral order. The convergence of the minimizing sequences is very good for cones with a small vertex half-angle. For $\theta > 45^\circ$, the best convergence is obtained with a cylindrical system of coordinate functions. /13

Figure 9 shows the values of ω_1^2, $\frac{\lambda_1^2}{\mu_1}$, $\frac{\lambda_{01}^2}{\mu_1}$ calculated by the variational method for two systems of coordinate functions. The dots indicate the exact

values for $\theta = 45^{\circ}$, obtained by Ye.Levin, and the experimental values for $\theta = 50^{\circ}$. The method of inscribed cylinders gives unsatisfactory results for conical cavities.

It should be noted that the frequencies obtained by Laurentz in combining the variational method with the long-wave theory are in satisfactory agreement with the more exact values only in the range of $\theta > 60^{\circ}$, while considerable discrepancies occur outside of this range.

<u>Torus.</u> All hydrodynamic factors for a toroidal cavity were determined by the variational method, using a cylindrical system of coordinate functions. The depth of the liquid and the ratio of the minimum inside radius to the maximum outside radius of the torus $\bar{h} : \delta$ were used as parameters. The calculation showed rapid convergence of the minimizing sequences, already in the third to fourth approximation.

Figure 10 shows the coefficients ω_1^2, $\frac{\lambda_1^2}{\mu_1}$, $\frac{\lambda'^2_1}{\mu_1}$ for a torus in the case of $\delta = 0.36$, calculated by the variational method, by the long-wave theory, and by use of coaxial cylinders of equivalent depth and peripheral radius, /14
forming the perimeter of the free surface. The graph indicates that the agreement of theory with experiment is entirely satisfactory over the entire range of depths.

3. <u>Study of Stability</u>

In the selection of the characteristics for the stabilization system, two problems must be solved: 1) determination of the requirements made by the object on the stabilization system; 2) investigation of the stability of the system as a whole.

In solving the first problem, we may consider a simplified system of equations, neglecting, for example, all nonlinear terms. Let a system of equations of the type of eqs.(2) describe the disturbed motion of a body with two cavities. An object will be called stabilizable if the stability of the closed system requires the control operator to ensure simultaneously, on both natural frequencies corresponding to the oscillations of the liquid in the cavities, either a phase advance or a phase lag. In the case where a phase lag on one frequency and a phase advance on the other is necessary for ensuring stability of the closed system, i.e., a condition often very difficult to realize in practice, the object will be termed unstabilizable. We note that the term "stability", as used here, means local dynamic stability, i.e., stability of a linear system of equations with "frozen" coefficients. The properties of stabilizability of an object are determined only by its parameters. Frequency methods may be applied to the investigation of problems of local dynamic stability.

Figure 11a shows the frequency characteristics of an open-loop system /15
for the case of a stabilizable object and Fig.11b, for the case of an unstabilizable object. Using the Nyquist criterion, it can be shown that in the

former case, when either phase advance or phase lag is established by a control operator on frequencies corresponding to the oscillations of the liquid, it is in principle possible to ensure stability of a closed-loop system, whereas in the latter case it is in principle impossible. This corresponds to the practical case where the frequencies of the oscillations of the liquid in the tanks are rather close.

Let the object have two cylindrical cavities of the same diameter. We introduce the following dimensionless parameters: $k = \frac{\rho_1}{\rho_2}$, $Z_1 = \frac{R_1}{\ell}$, $Z_2 = \frac{R_2}{\ell}$, $Z_3 = \frac{x_c - x_d}{\ell}$ where k is the density ratio of the liquids; R_1 and R_2 are the distances from the metacenter to the level of the liquid surface in the first and second tanks, respectively; $x_c - x_d$ is the distance from the metacenter to the point of application of the control forces; $\ell = \sqrt{\frac{J^o + J}{m^o + m}}$ is the radius of gyration (Fig. 11b).

Using the Hurwitz criterion, the following criterion of stabilizability can be obtained after several transformations:

Let

$$\Phi = (Z_2 + K Z_1)\left[(Z_2 + p)^2 + K(Z_1 + p)^2 - (1+K)a^2\right],$$

$$p = \frac{1 - Z_3^2}{2Z_3}, \qquad a = \frac{1 + Z_3^2}{2Z_3}.$$

If, at the instant of time t_1, the dimensionless parameters assume the values of $Z_1(t)$, $Z_2(t)$, $Z_3(t)$, then the object will be stabilizable if

$$\Phi\left[Z_1(t_1), Z_2(t_1), Z_3(t_1)\right] < 0$$

and unstabilizable if

$$\Phi\left[Z_1(t_1), Z_2(t_1), Z_3(t_1)\right] > 0.$$

Obviously, if the value of Z_3 is fixed, the boundary of the region of /16
stabilizability ($\Phi = 0$) of the plane (Z_1, Z_2) will consist of a straight line and an ellipse. Figure 12 shows this boundary; the region of unstabilizability is hatched. Similar regions may be constructed for various values of Z_3. The criterion of stabilizability for an object with tanks of other configurations can be obtained in the same manner. In this case, naturally, new design parameters must be added to the parameters k, Z_1, Z_2, Z_3.

Numerous nomograms of this type are now available for cavities of various shapes, permitting a very simple determination of the stabilizability of an object.

One of the methods of solving the second problem is to simulate the system of equations of disturbed motion of the object with a real device, on an analog computer. This method makes it possible to select the basic parameters of a stabilization system and to investigate the stability of the system as a whole, under various operating conditions. If the mathematical analog of a stabilization system can be constructed, then a computer can be used in investigating stability. In this case, the system of equations of disturbed motion is usually transformed into a system of second-order oscillators connected by the control operator. Another possibility is the method of harmonic averaging developed by N.N.Bogolyubov and Yu.A.Mitropol'skiy which yields differential equations for the envelope oscillation amplitudes. In this method, the envelope equations can be constructed by means of an alignment chart of amplitude-phase characteristics of the control operator, as a function of the amplitude of the input signal. The resultant equations for the envelopes may be interpreted by the numerical method.

In conclusion, we wish to indicate an interesting possibility of inves- /17
tigating the local dynamic stability, based on the use of mechanical (physical) models of the objects. In particular, closed-loop systems may be studied by this method. It requires the linking of the mechanical model to an electronic model consisting of an analog computer.

Figure 13 is a schematic diagram of the simulation of a closed-loop system. The dynamic properties of the object are fed by means of a mechanical model (MM) built in accordance with the criteria of similitude. The right-hand sides of the equations of disturbed motion as functions of the coordinates and of time, together with the control operator, are set up on the electronic model (EM), whose input is fed with signals from the position and velocity pickups (DD) of the mechanical model. The analogs of the forces and moments, generated in the electronic model, are transferred to the mechanical model by means of special force generators (CB) in the form of the corresponding forces and moments.

The mechanical model is freely suspended on cables or by other means. In most practical cases, the standard building blocks of analog computers can be used as the electronic model. The force generators may be either of the electrodynamic type with a fixed base, or of the jet pneumatic relay type. The latter do not require a fixed base and greatly simplify maintenance of the system as a whole.

Such an electromechanical model permits the most complete consideration of all features of the real mechanical system, including the smallest dissipative and nonlinear effects.

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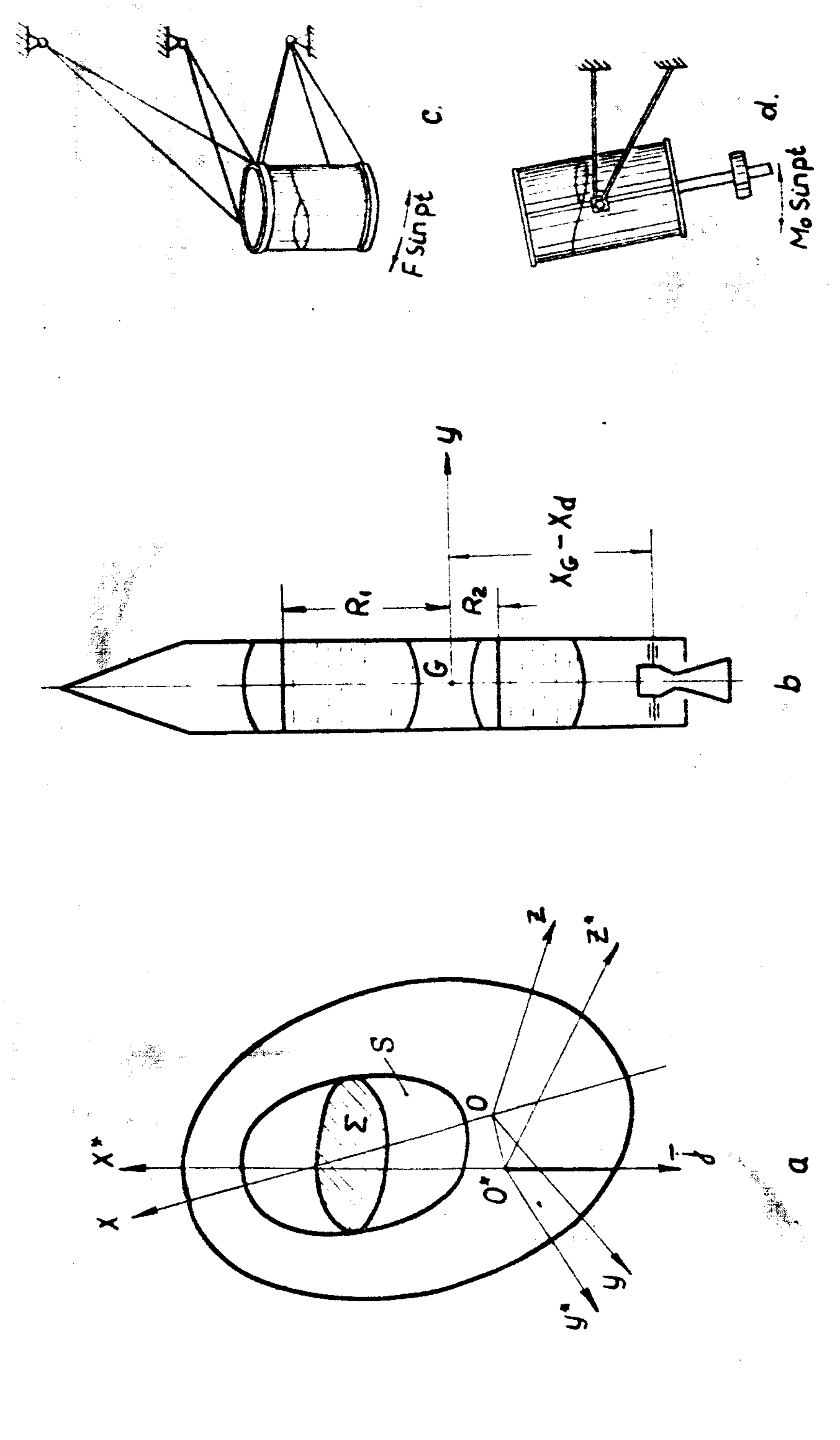


Fig.1

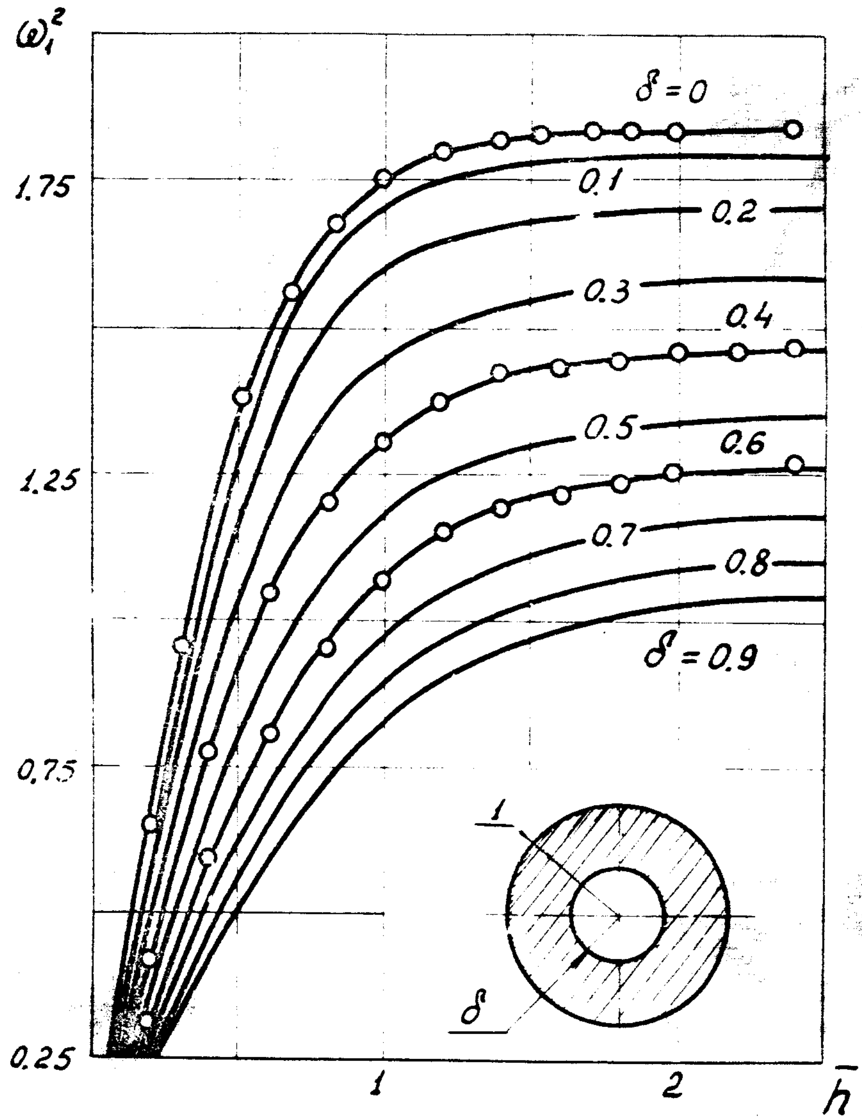

ω_1^2
1.75
1.25
0.75
0.25
$\delta = 0$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$\delta = 0.9$
1
2
$\bar{h}$
1
δ

Fig.2

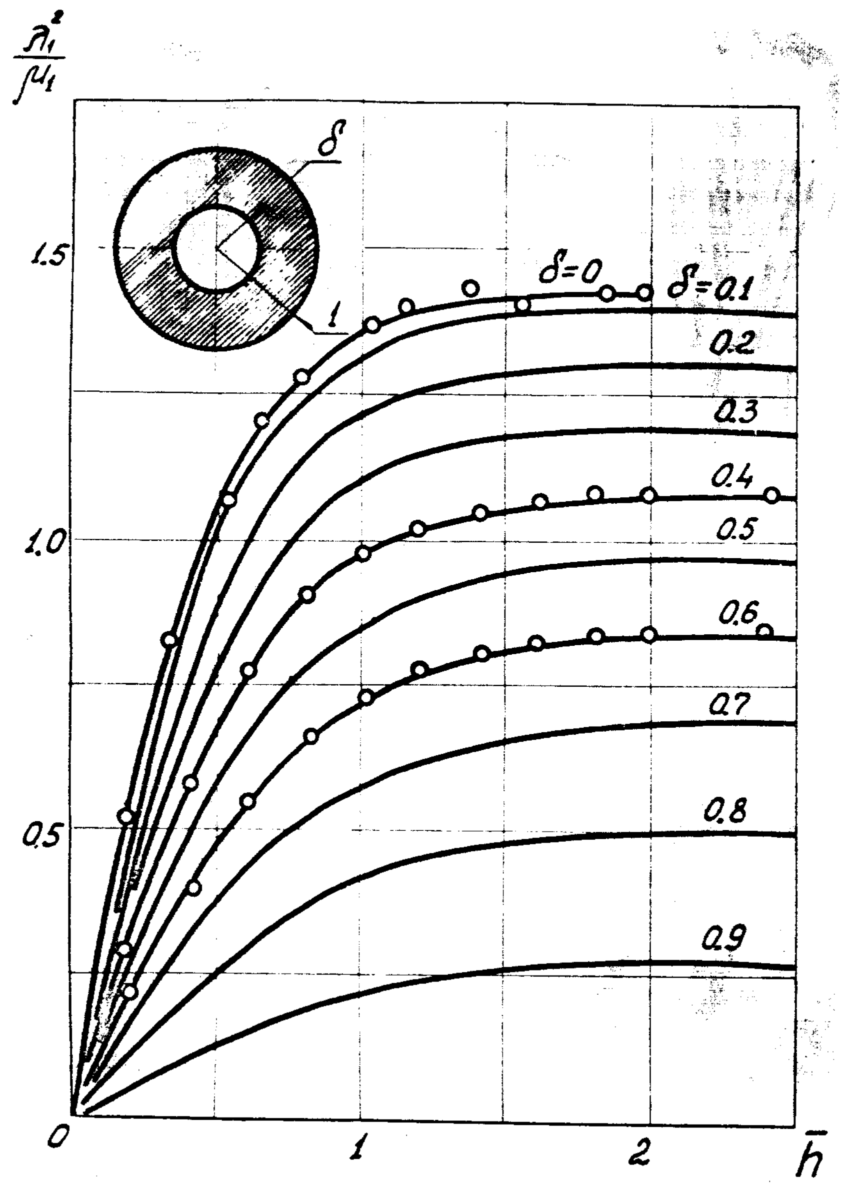
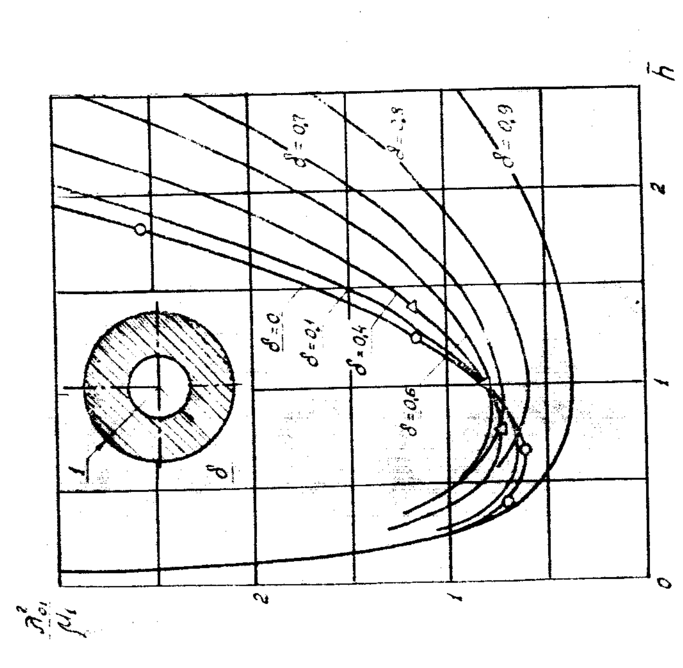
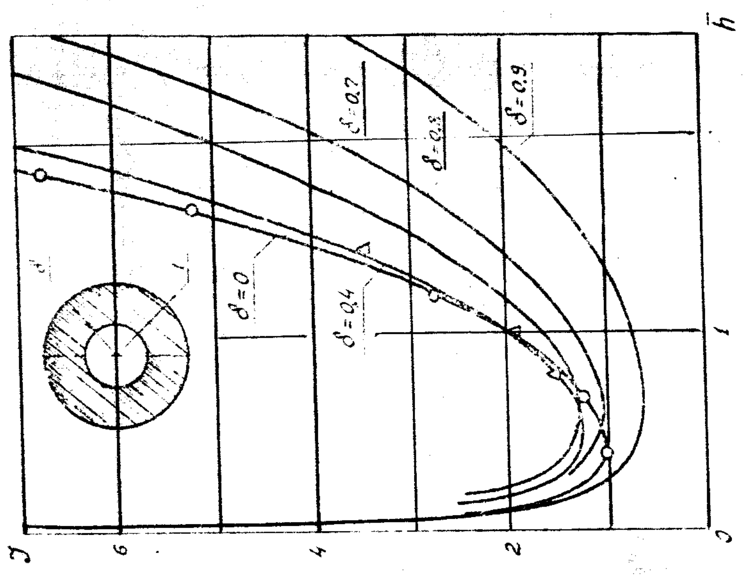

$\frac{\lambda_1^2}{\mu_1}$
1.5
1.0
0.5
0
1
2
$\bar{h}$
δ
1
δ=0
δ=0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

Fig.3



$\frac{\lambda_{01}^2}{\mu_1}$
2
1
0
1
2
$\bar{h}$
δ
1
δ=0
δ=0,1
δ=0,4
δ=0,6
δ=0,7
δ=0,9

Fig.4



J
6
4
2
0
1
$\bar{h}$
δ=0
δ=0,4
δ=0,7
δ=0,8
δ=0,9

Fig.5

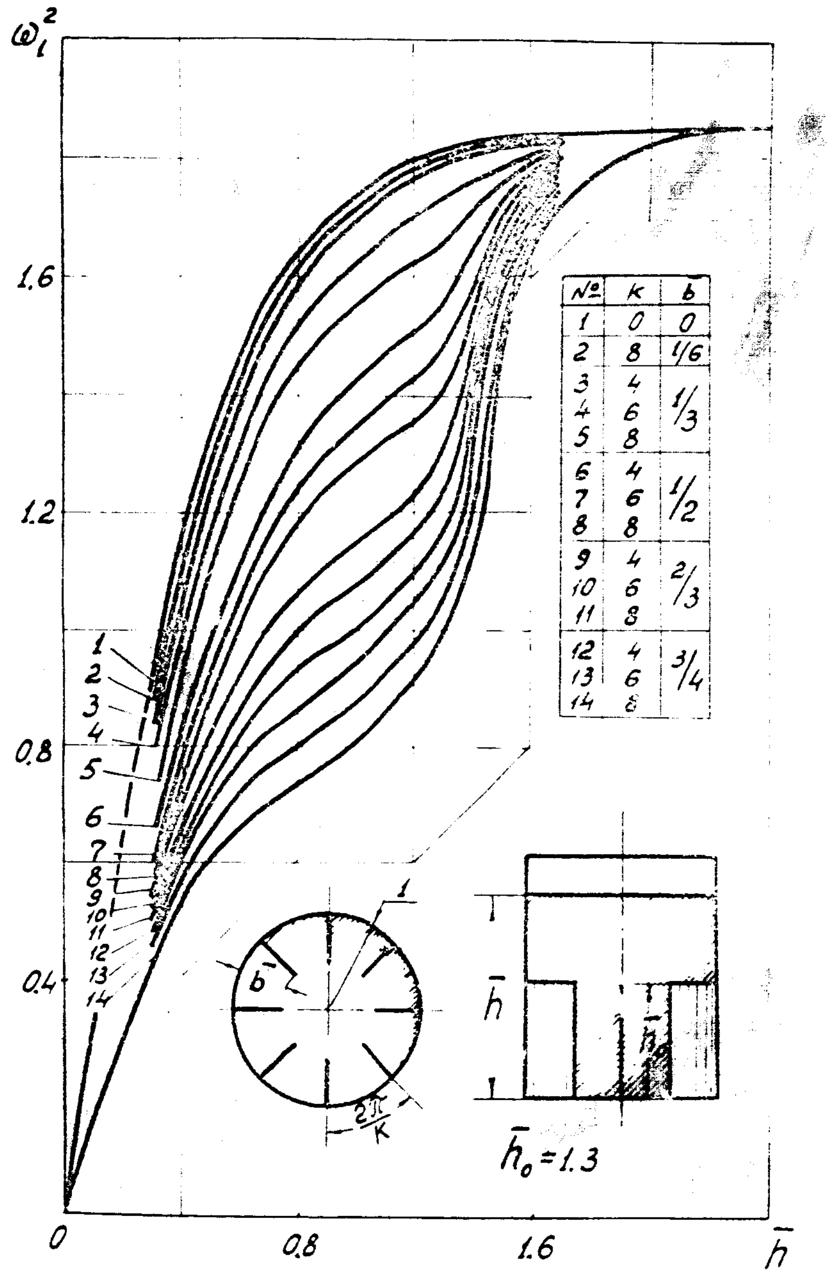

ω
1.6
1.2
0.8
0.4
0
0.8
1.6
h
1
2
3
4
5
6
7
8
9
10
11
12
13
14
№ K b
1 0 0
2 8 1/6
3 4
4 6 1/3
5 8
6 4
7 6 1/2
8 8
9 4
10 6 2/3
11 8
12 4
13 6 3/4
14 8
b
2π/K
h₀ = 1.3

Fig.6

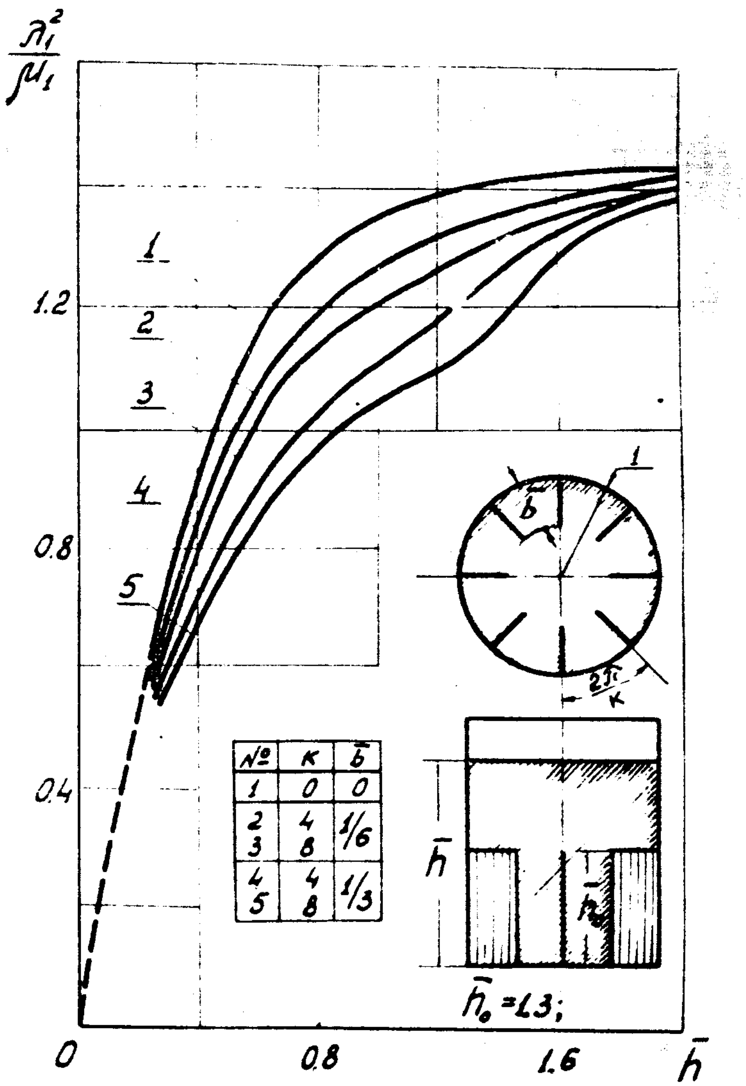
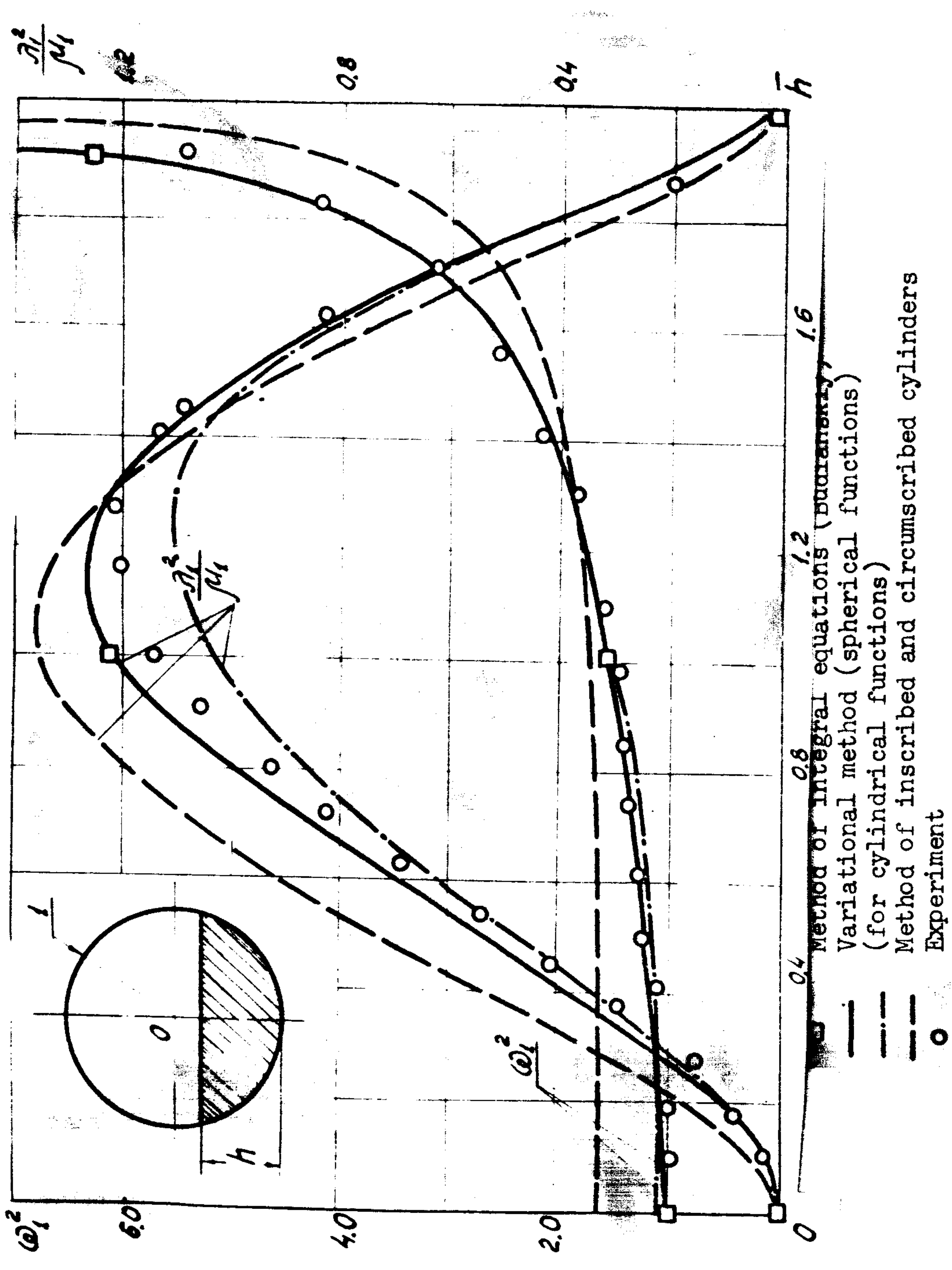
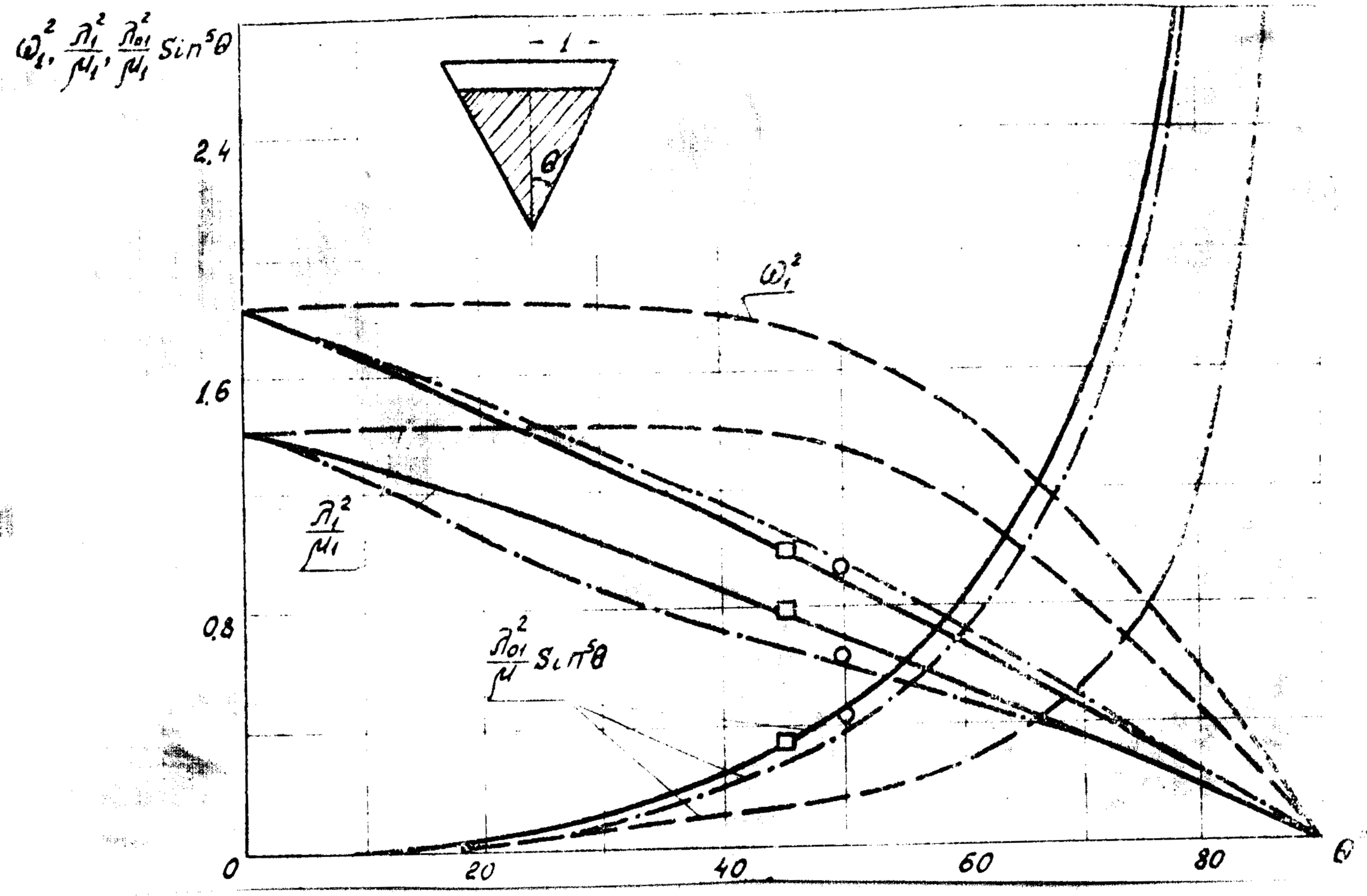

$\frac{\lambda_1^2}{\mu_1}$
1
2
3
4
5
1.2
0.8
0.4
0
0.8
1.6
$\bar{h}$
$\bar{b}$
1
$\frac{2\pi}{K}$
$\bar{h}$
$\bar{h}_0$
$\bar{h}_0 = 1.3;$
№ | K | $\bar{b}$
1 | 0 | 0
2, 3 | 4, 8 | 1/6
4, 5 | 4, 8 | 1/3

Fig.7



ω_1^2
6.0
4.0
2.0
0
0.4
0.8
1.2
1.6
$\bar{h}$
$\frac{\lambda_1^2}{\mu_1}$
1.2
0.8
0.4
1
h
Method of integral equations (Budiansky)
Variational method (spherical functions)
(for cylindrical functions)
Method of inscribed and circumscribed cylinders
Experiment

Fig.8



□ Exact solution (Ye.Levin)
Variational method (spherical functions)
(cylindrical functions)
Method of circumscribed cylinder
o Experimental data

Fig.9